Decoherence in current induced forces: Application to adiabatic quantum motors

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Current induced forces are not only related with the discrete nature of electrons but also with its quantum character. It is natural then to wonder about the effect of decoherence. Here, we develop the theory of current induced forces including dephasing processes and we apply it to study adiabatic quantum motors (AQMs). The theory is based on Büttiker's fictitious probe model, which here is reformulated for this particular case. We prove that it accomplishes the fluctuation-dissipation theorem. We also show that, in spite of decoherence, the total work performed by the current induced forces remains equal to the pumped charge per cycle times the voltage. We find that decoherence affects not only the current induced forces of the system but also its intrinsic friction and noise, modifying in a nontrivial way the efficiency of AQMs. We apply the theory to study an AQM inspired by a classical peristaltic pump where we surprisingly find that decoherence can play a crucial role by triggering its operation. Our results can help to understand how environmentally induced dephasing affects the quantum behavior of nanomechanical devices.

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I. INTRODUCTION

Nanomechanical devices in general and nanomotors in particular are topics that have attracted much attention in recent years [1–6]. The working principle of most of these experimental and theoretical proposals relies on classical physics. However, at nanoscale, one may benefit from the many phenomena emerging from quantum interferences. In this direction, reverse quantum pumping has been recently proposed as the basic mechanism by which dc-current induced forces can drive a nanoscopic motor. Such a device is now known as "adiabatic quantum motor" (AQM) [7–10]. Despite the classical description of the motor's movement, one can profit from the quantum nature of current induced forces. Indeed, one can boost the efficiency of AQMs by exploiting the interferences present in the Thouless pump [7].

Many natural questions arise when one addresses current induced forces in nanomechanical devices. Will their quantum behavior survive under realistic nonideal situations? Which is the effect of environmentally induced decoherence and how can we model it? Will the effect of decoherence be always counterproductive for an AQM? Solving these issues will provide a better understanding of their working principles and ways to assess the feasibility of their experimental implementations.

In this work, the theory of current induced forces based on scattering matrices [11–13] is extended to include decoherent events. Our approach is based on a reformulation of Büttiker's fictitious probe model [14–16]. This allows us to address the effect of decoherence on nonequilibrium current induced forces, friction coefficients, and fluctuating forces with a focus on AQMs.

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II. THEORY

A. Langevin equation and current induced forces

In the nonequilibrium Born-Oppenheimer approximation, the dynamical degrees of freedom of a system are slow as compared to the electron dynamics. In this limit, we can treat the mechanical degrees of freedom as a classical field acting on the electrons. Since the movement of a motor is cyclic, one can reduce its many degrees of freedom to a single rotational coordinate, x. Hence one can describe the rotor's dynamics by the 1D Langevin equation

$$M\ddot{x} + \frac{dU}{dx} = F - \gamma\dot{x} + \xi, \tag{1}$$

where M is the mass of the rotor (or the moment of inertia) and U is some classical external potential that can also be introduced. The right-hand side of Eq. (1) accounts for the current induced forces where F is a mean adiabatic reaction force. The second term is a friction (dissipative) force where γ is the friction coefficient. The last term, ξ, accounts for the force fluctuations. These current induced forces have a quantum origin and practical expressions in terms of the scattering matrices are given in Refs. [11–13], which are our starting point.

Like in Brownian motion, the interaction of the electrons with the rotor gives rise to a dissipation mechanism and a fluctuating force. At equilibrium, the only nonvanishing contribution to the friction coefficient is the symmetric contribution $\gamma^{s,\mathrm{eq}}$ [11–13]. On the other hand, there is a fluctuating force $\xi(t)$ whose self-correlation D, defined by $\langle \xi(t)\xi(t')\rangle \approx D\delta(t-t')$, is assumed locally correlated in time. These quantities must be related by the *fluctuation-dissipation theorem* (FDT), i.e., they satisfy $D = 2K_BT\gamma^{s,\mathrm{eq}}$, where K_BT is the thermal energy. We will consider the forces only up to first order in eV and/or $\dot{x}$. That implies that only the equilibrium contributions to the friction coefficient and D will be taken into account.

B. Decoherence and null current condition

Decoherence is included by connecting the system to a fictitious voltage probe ϕ. The voltmeter condition imposes $I_\phi = 0$, which provides charge conservation, while the reservoir character of the fictitious probe ensures the loss of memory of the reinjected electrons. Strictly speaking, this also involves inelastic events that redistribute electron's energy. However, in a linear response, $eV \to 0$, inelasticity is just reduced to a stochastic dephasing of the wave function, i.e., decoherence [17].

The total current flowing at lead m is

$$I_m = I_m^{\mathrm{Bias}} + I_m^{\mathrm{Pump}} + \delta I_m, \tag{2}$$

where I_m^{Bias} is the nonequilibrium current caused by an infinitesimal bias eV, I_m^{Pump} is the pumped current due to a variation of x, and δI_m accounts for current fluctuations. The fictitious voltage probe method has proved useful to address a quite related problem, the quantum pumping with dephasing [18,19]. While there, fluctuations could be disregarded, here, we need to consider them explicitly as they affect the dynamics of the system.

The nonequilibrium current incoming to the system through the lead $m = L,\phi$, in a linear response limit and at low temperatures, is $I_m^{\mathrm{Bias}} = (e/2\pi\hbar)(\sum_{n\neq m} T_{nm}\delta\mu_m - \sum_{n\neq m} T_{nm}\delta\mu_n)$. Here, T_{nm} is the transmittance between leads m and $n = L,R,\phi$; and $\delta\mu_m = \mu_m - \mu_R$ is the chemical potential of m, taking μ_R as reference.

The pumped current I_m^{Pump} through a lead m is $I_m^{\mathrm{Pump}} = e(dn_m/dx)\dot{x}$, where e is the electron charge, $\dot{x}$ the velocity of the rotational parameter x, and the emissivity of the lead m is

$$\frac{dn_m}{dx} = \int \frac{d\varepsilon}{2\pi \mathrm{i}}\left(-\frac{\partial f}{\partial \varepsilon}\right)\mathrm{Tr}\left(\Pi_m \frac{dS}{dx}S^\dagger\right), \tag{3}$$

where f is the Fermi distribution, Π_m is a projector onto the lead m, and S is the scattering matrix [19,20]. By integration of I_L^{Pump} we obtain the pumped charge per cycle through lead L [18,19],

$$Q_L = e\oint\left(\frac{dn_L}{dx} + \frac{T_{L\phi}}{T_{L\phi}+T_{\phi R}}\frac{dn_\phi}{dx}\right)\delta x. \tag{4}$$

Current fluctuations contain both the nonequilibrium shot noise and the thermal noise. Both of them satisfy $\langle \delta I_m\rangle = 0$, and $\sum_m \delta I_m = 0$. However, at equilibrium, the shot noise vanishes and only thermal noise survives.

The null current condition $I_\phi = 0$ at lead ϕ directly imposes a condition to $\delta\mu_\phi$. The value of $\delta\mu_\phi$ that ensures current cancellation of Eq. (2) is

$$\delta\mu_\phi = \frac{1}{T_{L\phi}+T_{\phi R}}\left(T_{L\phi}\delta\mu_L - 2\pi\hbar\frac{dn_\phi}{dx}\dot{x} - \frac{2\pi\hbar}{e}\delta I_\phi\right). \tag{5}$$

Since the variation of x is slow with respect to the electronic dynamics, we can consider that $\delta\mu_\phi$ adapts instantaneously to satisfy $I_\phi = 0$ at all time.

C. Current induced forces in presence of decoherence.

By considering an infinitesimal bias, $\delta\mu_L = eV$, we can split the Fermi function into an equilibrium and an out-of-equilibrium contributions, $f_m = f_0 + \Delta f_m$. Then, the force generated by the electric current [11–13],

$$F = \sum_m \int \frac{d\varepsilon}{2\pi \mathrm{i}} f_m \mathrm{Tr}\left(\Pi_m S^\dagger \frac{dS}{dx}\right), \tag{6}$$

can be split as $F = F^{\mathrm{eq}} + \Delta F$. The equilibrium force, F^{eq}, is conservative and thus, it does not produce work. At low temperatures and for small eV, $\int(\cdot)\Delta f_m d\varepsilon \simeq (\cdot)\delta\mu_m$. Using Eqs. (3), (5), and (6), we calculate the nonconservative forces for systems without magnetic fields up to first order in eV and $\dot{x}$,

$$\begin{aligned}\Delta F = &\left(\frac{dn_L}{dx} + \frac{T_{L\phi}}{T_{L\phi}+T_{\phi R}}\frac{dn_\phi}{dx}\right)eV \\ &- 2\pi\hbar\frac{\dot{x}}{T_{L\phi}+T_{\phi R}}\left(\frac{dn_\phi}{dx}\right)^2 \\ &+ \frac{2\pi\hbar}{e}\frac{\delta I_\phi}{T_{L\phi}+T_{\phi R}}\frac{dn_\phi}{dx}.\end{aligned} \tag{7}$$

The first term on the right-hand side (RHS) of Eq. (7) is the nonequilibrium force, F^{ne}, whose second term within the parenthesis is the decoherent contribution. This force is responsible for the work of the system, which is obtained by evaluating $W = \oint (F^{\mathrm{eq}} + F^{\mathrm{ne}})dx$, yielding

$$W = \oint dx\left(\frac{dn_L}{dx} + \frac{T_{L\phi}}{T_{L\phi}+T_{\phi R}}\frac{dn_\phi}{dx}\right)eV. \tag{8}$$

Comparing Eqs. (8) and (4), we realize that the work in presence of decoherence is proportional to the pumped charge per cycle times the voltage's bias,

$$W = QV. \tag{9}$$

This relation, proved valid for coherent AQMs, can be thought as a signature of the Onsager's reciprocal relations [21], showing that the model is well behaved.

The second term on the right-hand side (RHS) of Eq. (7) is a force proportional to the velocity. Thus one can associate it with a dissipative force whose origin is purely due to decoherence. The resulting friction coefficient γ^ϕ is

$$\gamma^\phi = \frac{2\pi\hbar}{T_{L\phi}+T_{\phi R}}\left(\frac{dn_\phi}{dx}\right)^2. \tag{10}$$

This proves that decoherence inside the sample enables energy dissipation through an additional friction of the rotor [22,23].

The third term on the RHS of Eq. (7) accounts for the fluctuations on the force induced by decoherence. The self-correlation of the fluctuating force ξ_ϕ, can be defined as $\langle \xi_\phi(t)\xi_\phi(t')\rangle \approx D_\phi \delta(t-t')$. Thus

$$D_\phi = \left(\frac{2\pi\hbar}{e}\frac{1}{T_{L\phi}+T_{\phi R}}\frac{dn_\phi}{dx}\right)^2 S_\phi, \tag{11}$$

where S_ϕ, the spectrum power of the fluctuating current at the lead ϕ, is defined as $\langle \delta I_\phi(t) I_\phi(t')\rangle \approx S_\phi\delta(t-t')$. In deducing Eq. (11), we use that transmittances and emissivities are already mean values. The only contribution to the current fluctuation, which is nonvanishing at equilibrium is the thermal fluctuation or Nyquist-Johnson noise [24]. Thus S_ϕ

is characterized by

$$S_\phi = 2K_BT\frac{e^2}{2\pi\hbar}(T_{L\phi}+T_{\phi R}). \tag{12}$$

Replacing Eq. (12) into Eq. (11) yields

$$D_\phi = 2K_BT\gamma^\phi, \tag{13}$$

which demonstrates that decoherent friction and fluctuating forces induced by decoherence are related by the FDT in our model as they should be.

D. Efficiency of AQMs

The thermodynamic efficiency, η_{TD}, can be defined as the useful output power that can be extracted from the system over the total input power. The useful output power is the work per cycle of the motor minus the energy lost due to friction, all divided by the period τ, i.e., $QV/\tau - \int_0^\tau \gamma\dot{x}^2dt/\tau$. The total input power is $IV + QV/\tau$. Then,

$$\eta_{\rm TD} = \frac{Q - 4\pi^2\gamma^*/(\tau V)}{\bar{g}\tau V + Q}. \tag{14}$$

Here, we have introduced $I = \bar{g}V$, where $\bar{g}$ is the average conductance, and the corrected average friction coefficient $\gamma^* = d^2\bar{\gamma}$, where $\bar{\gamma}$ is the average friction coefficient $\bar{\gamma} = \oint\gamma\dot{x}^2dt/\oint\dot{x}^2dt$. The dynamical constant d essentially accounts for the deviations of $\dot{x}$ with respect to its mean value, $d = \tau/(2\pi)\sqrt{\langle\dot{x}^2\rangle}$, where $\langle\dot{x}^2\rangle = \oint\dot{x}^2dt/\tau$.

From Eq. (14), one can extract the minimum energy necessary for $\eta \neq 0$, which is the minimum energy necessary to start the motor's motion $QV > 4\pi^2\gamma^*/\tau$. Additionally, one can also realize that there is an optimal value of τ that maximizes the efficiency,

$$\tau_0 = \frac{4\pi^2\gamma^*}{QV}\left(1+\sqrt{1+\frac{Q^2}{4\pi^2\gamma^*\bar{g}}}\right). \tag{15}$$

This value can be used to find the optimal load that the motor can move or, given a load, the optimal voltage to be applied to maximize the efficiency. Note that, when the average conductance goes to zero, τ_0 goes to infinity, which is consistent with the adiabatic limit of the efficiency proposed in Ref. [7].

III. RESULTS

Just to illustrate our theory, we will consider a simple example, see Fig. 1, of a quantum dot that while rotating changes its resonant energy, $E(\theta) = E_0 + \Delta E\cos(\theta + \theta_0)$, and the coupling to one reservoir, $V_R + \Delta V\sin(\theta)$. The angular coordinate is θ, with $\theta_0 = \theta(t=0)$. The couplings to the other reservoir L and to the fictitious probe ϕ are assumed constant. The system-environment interaction rate $2\Gamma_\phi/\hbar$ is determined by the coupling to the fictitious probe. The details of the solution of this example can be found in Appendix A. The main assumptions used are the following. (1) the Landauer-Büttiker picture of noninteracting electrons is valid. (2) The interaction with the leads can be taken within the wide band limit (WBL). (3) The interaction between the dot and the movable part of the system is perturbative, i.e., the variations of ΔE and ΔV are small with respect to Γ_0, the width of the dot's resonance without decoherence at $\theta = 0$. (4) The terminal velocity of the rotor in the stationary regime is approximately constant, i.e., $d \approx 1$. We briefly discuss the conditions for the validity of this last in Appendix B.

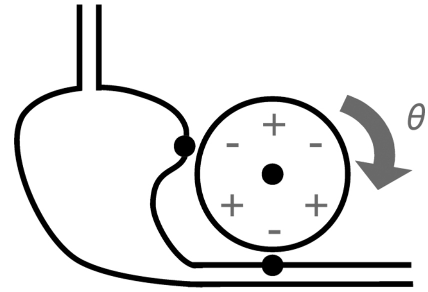

FIG. 1. Scheme of the type of system studied. A rotational device interacting with a quantum dot. Rotation of the motor changes the dot's energy as well as its coupling to one lead.

The work of the motor can be split into a coherent and a decoherent contributions. The behavior of W is similar to that described in Ref. [19] for the (decoherent) pumped charge. Details of the solution can be found in Appendix A, but the main results are shown in Fig. 2. In the on-resonance regime, i.e., when the Fermi energy ε is $\varepsilon \approx E_0$, the coherent contribution to the work is a monotonic decreasing function of Γ_ϕ, whereas the decoherent term increases with Γ_ϕ until it reaches a maximum value to decay afterward. The total work is always a decreasing function of Γ_ϕ. In the off-resonance regime, the coherent and the decoherent contributions behave

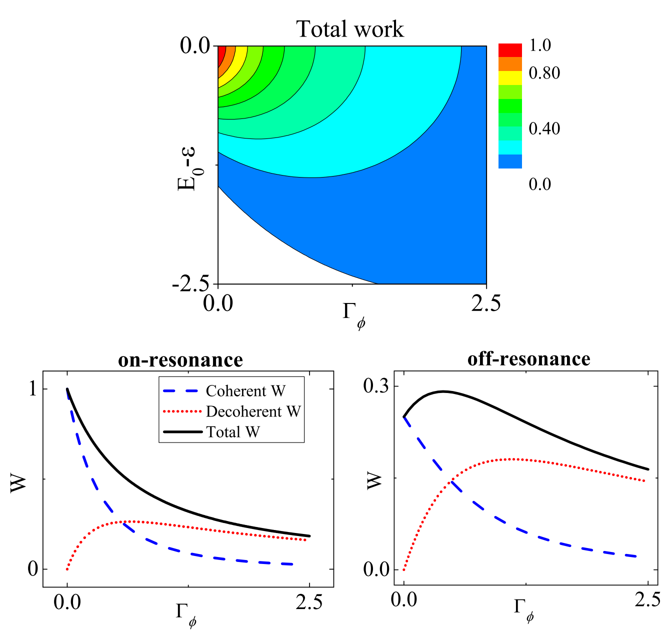

FIG. 2. (Color online) (Top) Total work of the AQM as function of $(E_0 - \varepsilon)$ and Γ_ϕ, both expressed in units of Γ_0. The work is normalized to its maximum value in the figure. (Bottom) Work of the AQM for the "on-" and "off-"resonance conditions, $(E_0 - \varepsilon) = 0$ and -1, respectively. Different contributions are marked in the inset. In all figures, the work is normalized to the maximum value of the total work.

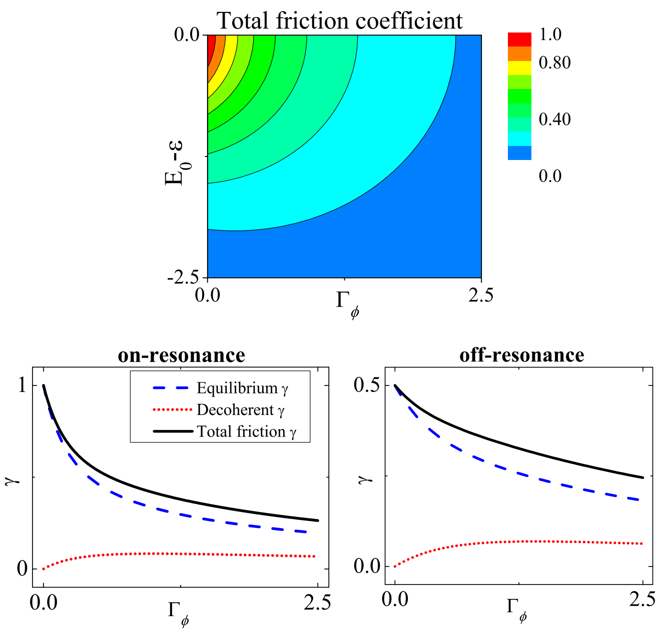

FIG. 3. (Color online) (Top) Total friction coefficient normalized to its maximum value as function of E_0 and Γ_ϕ, both expressed in units of Γ_0. (Bottom) Friction coefficient of the AQM for the "on-" and "off-"resonance conditions, $(E_0 - \varepsilon) = 0$ and -1, respectively. Different contributions are marked in the inset. In all figures, the values of the friction coefficients are normalized to the maximum value of the total friction coefficient.

qualitatively as in the on-resonance regime. However, here the decoherent term dominates and, then, the total work presents a maximum for a finite value of Γ_ϕ. This implies that in the off-resonance regime an adequate environment interaction can indeed maximize the work of the motor.

The friction coefficients $\bar{\gamma}^{s,\text{eq}}$ and $\bar{\gamma}^{\phi}$ present a similar behavior both in the on-resonance and in the off-resonance regimes as function of Γ_ϕ, see Fig. 3. $\bar{\gamma}^{s,\text{eq}}$ is a monotonically decreasing function of Γ_ϕ while $\bar{\gamma}^{\phi}$ presents a maximum. The total friction coefficient $\gamma = \gamma^{s,\text{eq}} + \gamma^{\phi}$ always decays with Γ_ϕ, both in the on-resonance and in the off-resonance regimes. In principle, just by looking Eqs. (7) and (10), one could (naively) expect that electronic friction is increased due to the extra friction term. However, electronic friction is actually mitigated by decoherence. This is because decoherence diminishes the quantum fluctuations and consequently also the friction coefficient, due to the FDT.

The effect of decoherence on the thermodynamic efficiency is complex due to a competition between the total work per cycle and the friction coefficient. In Fig. 4, we plot the efficiency as function of Γ_ϕ and the period τ. As predicted by Eq. (15), there is a (decoherence dependent) optimal value of τ that maximizes the efficiency. In this particular example the maximum efficiency is quite low, 1.610^{-5}. This is expectable considering the perturbative interaction between the rotor and the dot, which implies a small Q, and the high coupling between the dot and the reservoirs, which gives a high I^{Bias}. Beyond that, we find that, in the on-resonance regime, decoherence always diminishes the efficiency of the AQM. On the contrary, in the off-resonance regime, there is

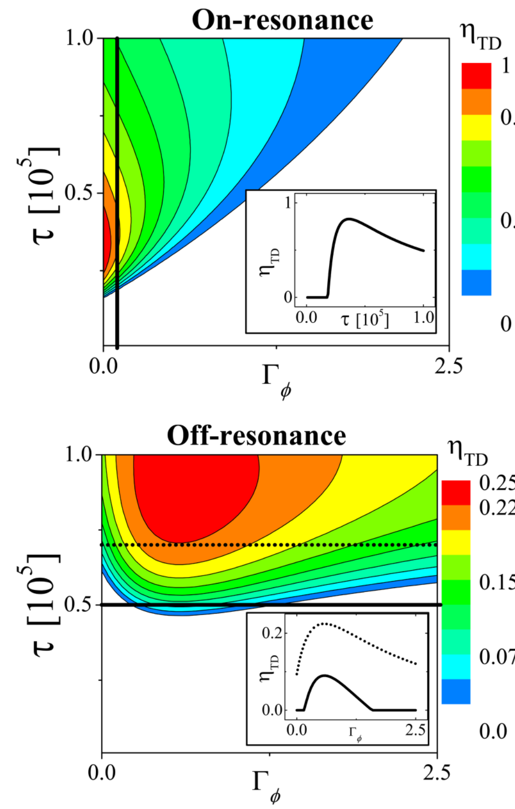

FIG. 4. (Color online) Thermodynamic efficiency of the AQM shown in Fig. 1 as function of the decoherent rate Γ_ϕ (in units of Γ_0) and the period of the motor τ (in units of $\hbar/2\Gamma_0$). (Top) On-resonance regime. (Bottom) Off-resonance regime $(E_0 - \varepsilon = -0.75\Gamma_0)$. The efficiency is normalized to its maximun value in all plots. Insets show the cuts marked in the main figures as continuous or dot lines.

a maximum in the efficiency for a finite value of Γ_ϕ. This shows that, surprisingly, efficiency can be increased due to a properly tunned interaction with the environment. Indeed, there are certain values of τ where at a critical Γ_ϕ the efficiency switches from zero to finite values. This implies that the AQM is indeed triggered by its interaction with environment.

IV. CONCLUSIONS

In conclusion, we developed the general theory of decoherent current induced forces and we applied it to AQMs. We showed that decoherence not only modifies the current induced forces but also the electronic or intrinsic dissipation mechanisms and its related force fluctuations [22,23]. We proved that the theory is consistent with the fluctuation-dissipation theorem. Besides, we showed that the relation between the total work and the pumped charge per cycle, proved for coherent AQMs [7], remains valid in presence of decoherence. We exemplified our theory with a simple example of AQMs showing that even there the role of decoherence can be nontrivial. Indeed, we showed that decoherence can increase the efficiency of the AQM. This may allow the design of novel devices such as environmentally activated nanomotors.

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APPENDIX A: DETAILS OF THE CALCULATION OF THE EXAMPLE OF AN AQM WITH DECOHERENCE

In this Appendix, we give the details of the calculations of the example presented in the main text, an AQM based on a peristaltic pump. The energy level of this AQM depends on θ, the rotational coordinate, as $E(\theta) = E_0 + \Delta E \cos(\theta + \theta_0)$, where θ_0 is a phase shift. The coupling to the reservoir R is $V_R + \Delta V_R \sin(\theta)$. We assumed a noninteracting electron picture. All the solutions are obtained considering a perturbative interaction between electrons and the mechanical degree of freedom, i.e., ΔE and ΔV_R are small with respect to Γ_0, the width of the dot's resonance without decoherence at $\theta = 0$. The chemical potential of the right reservoir is taken as the reference energy and we assumed a wide band approximation. All observables are evaluated at low temperatures and in a linear response regime, where $eV \to 0$. All this implies $\varepsilon = \mu_R$ in all the equations.

The effective Hamiltonian of the whole system, i.e., dot plus leads, reads

$$H(\theta) = E(\theta) + \Sigma_L + \Sigma_R(\theta) + \Sigma_\phi, \tag{A1}$$

where Σ_i is the self-energy of the lead $i = L,R,\phi$. We consider the leads within the wide band limit (WBL) where the self-energies Σ's are pure imaginary quantities and are independent of the Fermi energy ε. Thus $\Sigma_L = -\mathrm{i}\Gamma_L$, $\Sigma_\phi = -\mathrm{i}\Gamma_\phi$, and $\Sigma_R(\theta) = -\mathrm{i}\Gamma_R(\theta) = -\mathrm{i}[V_R + \Delta V_R \sin(\theta)]^2/V_{\text{BW}}$, where $4V_{\text{BW}}$ is the bandwidth of the lead. Here, the quantities Γ_L and Γ_ϕ are constants since the couplings to their corresponding leads are assumed independent of θ. The retarded Green function, defined as $G_0 = [\varepsilon - H(\theta)]^{-1}$, is

$$G_0 = \frac{1}{\varepsilon - (E(\theta) - \mathrm{i}\Gamma_L - \mathrm{i}\Gamma_R(\theta) - \mathrm{i}\Gamma_\phi)}. \tag{A2}$$

Transmittances $T_{m,n}$, needed to calculate the observables from Eqs. (4) to (8), can be obtained from

$$T_{m,n} = 2\Gamma_m |G_{m,n}|^2 2\Gamma_n, \tag{A3}$$

where $m \neq n$, and $m,n = L,R,\phi$. In this example, $G_{m,n} = G_0$, for all m,n. We can also obtain the density of states (DOS), $N(\varepsilon,\theta) = -(1/\pi)\mathrm{Im}(G_0)$, as

$$N(\varepsilon,\theta) = \frac{1}{\pi}\frac{\Gamma(\theta)}{(\varepsilon - E(\theta))^2 + \Gamma(\theta)^2}, \tag{A4}$$

where $\Gamma(\theta) = \Gamma_L + \Gamma_\phi + \Gamma_R(\theta)$. The complete S matrix can also be obtained from (A2):

$$\mathbb{S} = \mathbb{I} - 2\mathrm{i}G_0 \begin{pmatrix} \Gamma_L & \sqrt{\Gamma_L\Gamma_R(\theta)} & \sqrt{\Gamma_L\Gamma_\phi} \\ \sqrt{\Gamma_L\Gamma_R(\theta)} & \Gamma_R(\theta) & \sqrt{\Gamma_R(\theta)\Gamma_\phi} \\ \sqrt{\Gamma_L\Gamma_\phi} & \sqrt{\Gamma_R(\theta)\Gamma_\phi} & \Gamma_\phi \end{pmatrix}. \tag{A5}$$

Thus the matrix $[S^\dagger \frac{dS}{d\theta}]$ gives

$$S^\dagger \frac{dS}{d\theta} = -2\mathrm{i}|G_0|^2 \Lambda, \tag{A6}$$

where the elements of the operator Λ are

$$\begin{aligned}
\Lambda_{1,1} &= \Gamma_L \frac{dE(\theta)}{d\theta}, \\
\Lambda_{1,2} &= \sqrt{\Gamma_L\Gamma_R(\theta)}\frac{dE(\theta)}{d\theta} \\
&\quad + \sqrt{\frac{\Gamma_L}{4\Gamma_R(\theta)}}(\varepsilon - E(\theta) + \mathrm{i}\Gamma(\theta))\frac{d\Gamma_R(\theta)}{d\theta}, \\
\Lambda_{1,3} &= \sqrt{\Gamma_L\Gamma_\phi}\frac{dE(\theta)}{d\theta}, \\
\Lambda_{2,2} &= \Gamma_R(\theta)\frac{dE(\theta)}{d\theta} + (\varepsilon - E(\theta))\frac{d\Gamma_R(\theta)}{d\theta}, \\
\Lambda_{2,3} &= \sqrt{\Gamma_R(\theta)\Gamma_\phi}\frac{dE(\theta)}{d\theta} \\
&\quad + \sqrt{\frac{\Gamma_\phi}{4\Gamma_R(\theta)}}(\varepsilon - E(\theta) + \mathrm{i}\Gamma(\theta))\frac{d\Gamma_R(\theta)}{d\theta}, \\
\Lambda_{3,3} &= \Gamma_\phi \frac{dE(\theta)}{d\theta},
\end{aligned}$$

with $\Lambda_{m,n} = \Lambda^*_{n,m}$.

At this point, we can evaluate all the physical quantities that are relevant to this problem. Let us start with the emissivities of Eq. (3). At low temperatures, we can use $\partial f_m/\partial\varepsilon = -f_m(1 - f_m)/K_BT \simeq -\delta(\varepsilon - \mu_m)$. Thus we have

$$\frac{dn_m}{d\theta} = \frac{1}{2\pi\mathrm{i}}\mathrm{Tr}\left(\Pi_m S^\dagger \frac{dS}{d\theta}\right) \tag{A7}$$

$$= \frac{1}{2\pi\mathrm{i}}\left(S^\dagger \frac{dS}{d\theta}\right)_{m,m}, \tag{A8}$$

where $m = L,R,\phi$. Then

$$\begin{aligned}
\frac{dn_{L(\phi)}}{d\theta} &= -\frac{N(\varepsilon,\theta)}{\Gamma(\theta)}\frac{dE(\theta)}{d\theta}\Gamma_{L(\phi)}, \\
\frac{dn_R}{d\theta} &= -\frac{1}{\pi}\frac{N(\varepsilon,\theta)}{\Gamma(\theta)} \\
&\quad \times \left[\frac{dE(\theta)}{d\theta}\Gamma_R(\theta) + \frac{d\Gamma_R(\theta)}{d\theta}(\varepsilon - E(\theta))\right].
\end{aligned} \tag{A9}$$

We can insert these expressions into Eq. (4) to obtain the pumped charge per cycle or the total work, note that $W = QV$. The result can be split into a coherent and a decoherent contributions, $W = W^{\text{coh}} + W^{\text{dec}}$. The mathematical expression for W^{coh} is exactly the same with or without decoherence (however, its value depends on Γ_ϕ). Consistently, W^{dec} is zero in a purely coherent case.

By evaluating Eq. (8) with Eq. (A9) and using Green's theorem [20], we obtain

$$W = -eV(\Omega_{\text{coh}} + \Omega_{\text{dec}})\Delta E \Delta V_R \cos(\theta_0), \tag{A10}$$

where

$$\Omega_{\text{coh}} = \frac{\Gamma_L V_R}{\Gamma_T V_{\text{BW}}}(4\pi^2 N^2(\varepsilon)), \tag{A11}$$

$$\Omega_{\rm dec} = \frac{\Gamma_\phi \Gamma_L V_R}{\Gamma_0 \Gamma_T V_{\rm BW}} \left[\frac{2\pi N(\varepsilon)}{\Gamma_0} + 4\pi^2 N^2(\varepsilon) \right]. \quad \text{(A12)}$$

Here, $\Gamma_T = \Gamma(\theta)|_{\Delta V_R=0,\Delta E=0}$, which does not depend on θ, and $\Gamma_0 = \Gamma_L + V_R^2/V_{\rm BW}$. These equations are expressed at zero order, i.e., for small ΔE and ΔV_R. Note that the factor $\Delta E \Delta V_R \cos(\theta_0)$ is the parametric area enclosed in a cycle of the parameter θ.

The friction coefficient at equilibrium, $\gamma^{s,{\rm eq}}$, is

$$\gamma^{s,{\rm eq}} = \frac{1}{4} \sum_{m,n} \int \frac{d\varepsilon}{2\pi} \frac{\partial}{\partial \varepsilon}(f_m + f_n) \times {\rm Tr}\left(\Pi_m S^\dagger \frac{dS}{d\theta} \Pi_n S^\dagger \frac{dS}{d\theta} \right). \quad \text{(A13)}$$

Assuming a low-temperature limit to evaluate the integral, yields

$$\bar{\gamma}^{s,{\rm eq}} = \frac{\hbar}{8\pi^2} \oint \sum_{m,n} \left| \left(S^\dagger \frac{dS}{d\theta} \right)_{m,n} \right|^2 d\theta. \quad \text{(A14)}$$

The additional friction term, which is a direct consequence of decoherence on current induced forces, can be evaluated from Eq. (10), giving

$$\bar{\gamma}^\phi = \hbar \frac{\Gamma_\phi}{4\Gamma_0 \Gamma_T} N(\varepsilon) \Delta E^2. \quad \text{(A15)}$$

Notice that the positivity of both coefficients is guaranteed.

List of parameters. The parameters used in the whole work to perform the calculations are $eV = 10^{-4} V_{\rm BW}$, $V_R = \sqrt{0.1} V_{\rm BW}$, $\Gamma_L = 0.1 V_{\rm BW}$, $\Delta V_R = 10^{-2} V_R$, $\Delta E = 210^{-3} V_{\rm BW}$, and $\Gamma_0 = \Gamma_L + V_R^2/V_{\rm BW} = 0.2 V_{\rm BW}$. The maximum values of the efficiency, total work, and total friction coefficient used to normalize the plots are 1.610^{-5}, $10^{-4} eV$, and $2.410^{-5}\hbar$, respectively.

APPENDIX B: LIMIT OF CONSTANT TERMINAL VELOCITY

At steady state, all the energy that is absorbed by the motor is completely dissipated by friction and thus, the total energy is conserved according to

$$\int_0^{2\pi} [F(\theta) - F_{\rm load} - \gamma \dot{\theta}(\theta)] d\theta = 0, \quad \text{(B1)}$$

where $F_{\rm load}$ account for an external load to the system. Here, we are assuming an average over steady-state ensembles, so random forces can be neglected and we can use $\dot{\theta}(t) = \dot{\theta}(t+\tau) = \dot{\theta}(\theta)$.

From Eq. (B1) and using the same definitions for $\bar{\gamma}$ and $\langle \dot{\theta}^2(t) \rangle$ as in the main text, we obtain

$$\langle \dot{\theta}^2(t) \rangle = \frac{QV - W_{\rm load}}{\bar{\gamma}}, \quad \text{(B2)}$$

where $W_{\rm load}$ is the work done by the external forces in one cycle. Using this result, the mean kinetic energy becomes

$$\langle K \rangle = \frac{1}{2} I \langle \dot{\theta}^2(t) \rangle = \frac{1}{2} I \frac{(QV - W_{\rm load})}{\bar{\gamma}}, \quad \text{(B3)}$$

where I is the moment of inertia of the rotor.

The change of the kinetic energy due to a rotation of the parameter θ is $\Delta K(\theta) = -\Delta U^*(\theta)$, where $U^*(\theta) = -\int_0^\theta [F(\theta') - F_{\rm load} - \gamma \dot{\theta}(\theta')] d\theta'$ is a pseudopotential defined only at the steady state. When the kinetic energy of the motor is much greater than $\Delta U^*(\theta)$,

$$\frac{1}{2} I \frac{(QV - W_{\rm load})}{\bar{\gamma}} \gg \Delta U^*(\theta), \quad \text{(B4)}$$

then $\Delta K(\theta)/\langle K \rangle \to 0$ and, thus, the rotational velocity of the system becomes insensitive to θ. Therefore $\tau = \int_0^{2\pi} d\theta/\dot{\theta}(\theta) \approx 2\pi/\dot{\theta}$ and hence the dynamical factor $d = \frac{\tau}{2\pi}\sqrt{\langle \dot{\theta}^2(t) \rangle} \approx 1$. Note that the terminal velocity is independent of I. Therefore the condition given in Eq. (B4) is always valid in the limit of macroscopic rotors, that is, rotors with a sufficiently large moment of inertia.

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