Influence of ophthalmic lenses in the angle subtended by a peripheral source point

Influencia de lentes oftálmicas en el ángulo subtendido por un punto fuente periférico

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ABSTRACT:

When a myopic or hyperopic eye wearing an ophthalmic lens looks at a stimulus and also receives light from a peripheral point, the lens modifies the eccentricity angle with which light originated at this point reaches the eye. Under paraxial approximation, the lens visual magnification (quotient between the angles subtended at the corneal vertex by the point and by its image through the lens) can be evaluated as usual, either considering thick or thin lens. In the first case, the calculation requires data (not always available) concerning source location, lens construction parameters and vertex distance while, in the second, results are imprecise for thick lenses employed by patients suffering severe hyperopia. To avoid these drawbacks, in the present article we obtain a formula for the mean visual magnification of a standard ophthalmic lens of spherical surfaces as a function only of its power. We propose the use of this formula to estimate in a simple and approximate way the effective eccentricity angle corresponding to a far or near point source and to any standard ophthalmic lens and habitual vertex distance.

Key words: Ophthalmic Lens, Peripheral Point Source, Eccentricity Angle.

RESUMEN:

Cuando un ojo miope o hipermétrope utilizando una lente oftálmica mira un estímulo y, además, recibe luz de un punto periférico, la lente modifica el ángulo de excentricidad con el cual la luz originada en dicho punto llega al ojo. En aproximación paraxial, el aumento visual de la lente (cociente entre los ángulos subtendidos en el vértice corneal por el punto y por su imagen a través de lente) puede evaluarse como es usual, considerando o bien lente gruesa o delgada. En el primer caso, el cálculo requiere datos (no siempre disponibles) referentes a ubicación de la fuente, parámetros constructivos de la lente y distancia de vértice mientras que, en el segundo, los resultados son imprecisos para lentes gruesas empleadas por pacientes con hipermetropía severa. Para evitar estos inconvenientes, en el presente artículo obtenemos una formula para el aumento visual promedio de una lente oftálmica estándar de caras esféricas como función solo de su potencia. Proponemos utilizar esta fórmula para estimar de modo sencillo y aproximado el ángulo de excentricidad efectivo correspondiente a un punto fuente lejano o cercano y a cualquier lente oftálmica estándar y distancia de vértice habitual.

Palabras clave: Lente Oftálmica, Fuente Periférica, Ángulo de Excentricidad.

REFERENCES AND LINKS

[1] R.B. Rabbetts, Bennett & Rabbetts´Clinical Visual Optics, Butterworth-Heinemann, London (2007).

[2] D. A Atchison., E. L. Markwell, S. Kasthurirangan, J. M. Pope, G. Smith, P. G. Swann, "Age-related changes in optical and biometric characteristics of emmetropic eyes", *J. Vision* **8**, 1-10 (2008).

[3] J. Tabernero, A. Benito, V. Nourrit, P. Artal, "Instrument for measuring the misalignments of ocular surfaces", *Opt. Express* **14**, 10945-10956 (2006).

[4] L. Lundström, S. Manzanera, P. M.Prieto, D. B.Ayala, N. Gorceix, J. Gustafsson, P. Unsbo, P. Artal, "Effect of optical correction and remaining aberrations on peripheral resolution acuity in the human eye", *Opt. Express* **15**, 12654-12661 (2007).

[5] V. Viqueira Pérez, F. M. Martínez Verdú, D. de Fez Saiz, Óptica Fisiológica: Modelo Paraxial y Compensación Óptica del Ojo, Universidad de Alicante, Alicante (2003).

[6] R. Blendowske, E. A. Villegas, P. Artal, "An analytical model describing aberratins in the progression corridor of progressive addition lenses", *Optometry Vision Sci.* **83**, 666-671 (2006).

[7] G. A. Fry, M. Alpern, "The effect of a peripheral glare source upon the apparent brightness of an object", *J. Opt. Soc. Am*. **43**, 189-195 (1953).

[8] J. J. Vos, "Disability glare - A state of the art report", *CIE Journal* **3,** 39-53 (1984).

[9] L. Issolio, J. Matranga, P. Barrionuevo, S. A. Comastri, G. Martin, E. Colombo, "Brightness reduction as a function of the angular position of a glare source", *Opt. Pura Apl.* **42**, 33-39 (2009**).**

[10] L. Issolio, N. López Gil, E. Colombo, I. Miró, "Brightness perception under transient glare conditions with myopic subjects wearing contact lenses and spectacles", 4th Iberoamerican Meeting on Optics and 7th Latin American Meeting of Optics, Lasers and their Applications, Proceedings SPIE **4419**, 557-560 (2001).

[11] S. A. Comastri, J. Matranga, K. Bastida, L. Issolio, E. Colombo, "Influence of ophthalmic lenses in the illuminance originated at a peripheral glare source and reaching the eye", *Optik* **120**, 860-872 (2009).

[12] M. S. Millan, J. Escofet, E. Perez, Óptica Geométrica, Ariel, Barcelona (2004).

[13] R. S. Longhurst, Geometrical and Physical Optics, Longman, London (1973).

[14] E. A. Villegas, "Desviaciones prismáticas en lentes oftálmicas (II): efecto prismático efectivo", *Ver y Oir* **224**, 1-15 (2008).

1. Introduction

Visual performance [1] depends on each individual subject, on his age [2] and on his psychophysical state. The optical ocular system is a misaligned system [3] which can be limited by low and high order aberrations that vary according to the stimulus eccentricity angle [2,4]. The most common aberration is defocus and it can be corrected with refractive surgery, intraocular lenses, contact lenses and, most frequently, with ophthalmic lenses [5] which can be monofocal, bifocal or progressive (addition ranging from 0.75 D to 3.5 D [6]). When a subject wearing ophthalmic lenses looks at a point and receives light from a peripheral source point, the effective angle subtended by this point is not the same as without lenses. This can be important, for example, when analysing the effects of a transient peripheral glare source on vision of a foveal stimulus (this source could be the headlight of an oncoming car when night driving in routes). Due to this source, the perceived brightness of the foveal stimulus decreases and this can be accounted for in terms of a veiling luminance that is proportional to the illuminance reaching the eye and inversely proportional to the square of the eccentricity angle [7,8]. When ophthalmic lenses are worn, this illuminance and this angle are modified [9-11].

In the present paper, we consider a foveally fixated myopic or hyperopic eye wearing a centred standard monofocal ophthalmic lens (with its optical axis coinciding with the line of sight) and receiving light from a peripheral source point that subtends an angle θ at the corneal vertex. We estimate the influence of the ophthalmic lens in the angle subtended at the corneal vertex by the image of this point through the lens, henceforth termed effective eccentricity angle, θ_G. We consider angles subtended at the corneal vertex (intersection line of sight-cornea) because unlike other points [1,12-14] useful under visual conditions different from the one considered here, it is accessible from outside. In Section 2 we describe the configuration and our considerations concerning objects and lenses. In Section 3, we evaluate the effective angle and the corresponding visual magnification [12], $\gamma_G=\theta_G/\theta$, for both a distant and a near source and for a set of ophthalmic lenses and vertex distance termed “original”. Assuming the paraxial approximation is valid, first we consider the lenses to be thick and then thin. In Section 4, taking into account these results, we derive a formula for the mean visual magnification (Γ_G) only as a function of lens power and propose its use to estimate θ_G for any standard ophthalmic lens of spherical surfaces. In Section 5, as an example, we show an application of this formula.

2. Configuration and considerations

Henceforth superscripts *(M)* and *(H)* indicate myopic and hyperopic eyes while subscripts *r* and *p* indicate far and near source; *G* and LC magnitudes associated to ophthalmic and contact lenses and *t* indicates thin lens. Powers are measured in diopters, distances in millimetres and angles in degrees.

We consider that the stimulus is centred at a point Q at a distance s_Q from VC (Fig.1) and that the peripheral source point J is at a distance s_J (which can coincide or not with s_Q), J can be far (s_J=–∞), near (the typical [6] reading distance being s_J=–400 mm) or at an arbitrary distance from the eye.

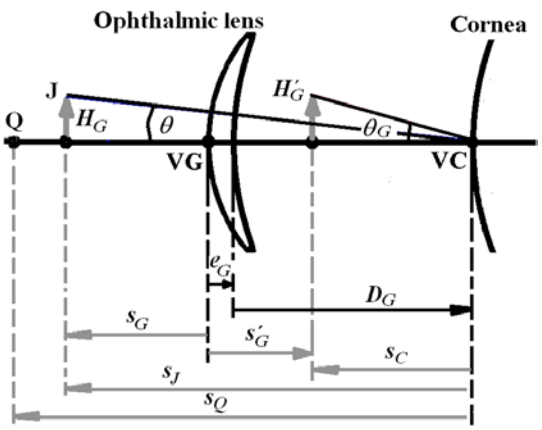

Fig. 1. Ophthalmic lens notation. VG and VC: vertexes; D_G: vertex distance; e_G: axial thickness; n_G: lens refraction index; H_G and H'_G: object and image transversal distances; s_G and s'_G: object and image distances measured from VG.

The power of each standard ophthalmic lens, Φ_G, is the one required by the ametrope to view the stimulus, Φ_G<0 for a diverging lens worn by a myopic eye and Φ_G>0 for a converging lens worn by a hyperopic eye. For a certain lens power, the construction parameters, this is, the curvature radii of the anterior and posterior surfaces (R_{G1} and R_{G2}), the refraction index (n_G) and the axial thicknesses (e_G), are not unique since various types of lenses are fabricated to meet different customers requirements. When the visual performance of a subject wearing a lens is analyzed, these parameters can be fabrication data, measured or unknown. Initially we consider an original set of 6 ophthalmic lenses of ±2, ±4 and ±6 D (Table I) and a vertex distance D_G=12 mm. We have [12,13] $\Phi_G=\Phi_{G,t}-\Phi_{\mathrm{Ad}}$ and $\Phi_{G,t}=\Phi_1+\Phi_2$ (with $\Phi_1=1000(n_G-1)/R_{G1}$ and $\Phi_2=1000(1-n_G)/R_{G2}$) and we obtain $\Phi_{\mathrm{Ad}}=\Phi_1\Phi_2 e_G/(1000 n_G)<0.25$ D.

TABLE I
Original set of ophthalmic lenses.

Φ_G (D)	R_{G1} (mm)	R_{G2} (mm)	e_G (mm)
-6.0	334.7	66.8	2.1
-4.0	270.6	85.6	1.2
-2.0	150.0	93.7	1.0
2.0	75.0	105.2	3.5
4.0	71.1	159.3	5.8
6.0	59.2	193.1	8.2

3. Paraxial angles and visual magnification

Angles θ and θ_G are represented in Fig. 2, the image of a real object through a diverging (or converging) lens is non-inverted (or inverted) and acts as a real (or virtual) object for the eye which, accordingly, yields an inverted (or non-inverted) image. Leaving aside aberrations, we find $\gamma_G=\theta_G/\theta$ under paraxial approximation considering thick and thin lens.

3.a. Paraxial results considering thick lens

For each ophthalmic lens constituting the original set, we consider D_G=12mm; θ=10º and the limiting cases in which the source is far (s_J=–∞) and near (s_J=–400 mm). Point J is such that $\theta=(H_G/s_J)(180^\circ/\pi)$ and $s_G=s_J+e_G+D_G$ (Fig. 1). At each lens surface we have, $n'/s'-n/s=(n'-n)/R$ and $m=H'/H=(n/n')(s'/s)$ (R being the surface curvature radius; s and s' object and image distances; n and n' refraction indices; H and H' object and image heights and m the lateral magnification) [12,13]. Evaluating the distances H'_G and s'_G corresponding to the image of J, we obtain $s'^{(M)}_G<0$ and $s'^{(H)}_G>0$ (Fig. 2). This image is at a distance $s_C=s'_G-D_G-e_G$ from VC and the effective angle is $\theta_G=(H'_G/s_C)(180°/\pi)$. Thus we calculate visual magnification □(Fig. 3) using the formula:

$$\gamma_G = \frac{\theta_G}{\theta} = \frac{s_J}{s_C}\frac{H'_G}{H_G}, \qquad (1)$$

and get $\gamma_G^{(M)}<1$ and $\gamma_G^{(H)}>1$ for far and near source.

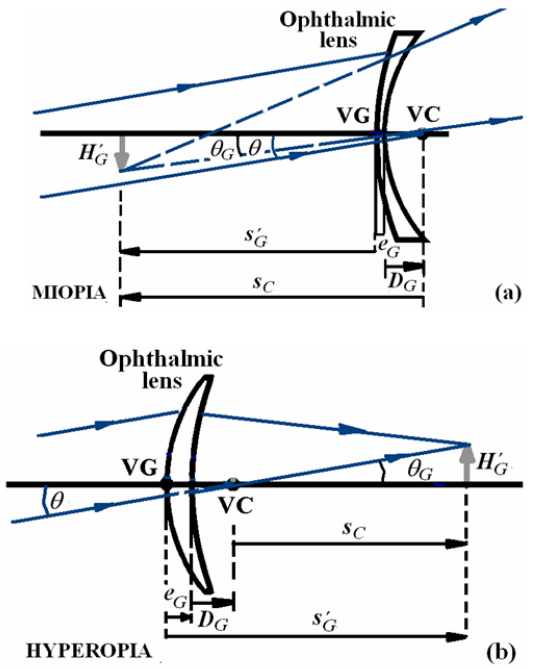

Fig. 2. Paraxial image through ophthalmic lenses.

3.b. Paraxial results assuming thin lens

Assuming the ophthalmic lens to be thin [12,13] we have $s'_G=s_G/(1+s_G\Phi_{G,t})$ and $H'_G/H_G=(s'_G/s_G)$. Since $s_J=s_G(1-D_G/s_G)$; $s_C=s'_G(1-D_G/s'_G)$ and $\Phi_G\approx\Phi_{G,t}$ (Fig.1), substituting in Eq.(1), visual magnification is $\gamma_{G,t}=1/(1\text{-}0.001D_G\Phi_G/(1-D_G/s_G)(\text{mm D})))$. Since $D_G/|s_G|<<1$ for cases of interest in this work and considering a 1st order Taylor expansion (subscript T) in $0.001D_G\Phi_G/(1-D_G/s_G)$, visual magnification for a thin lens and a 1st order Taylor expansion is

$$\gamma_{G,t,T} = 1 + 0.001\left(\frac{\Phi_G D_G}{1-D_G/s_G}\right). \qquad (2)$$

For D_G=12mm, the plot $\gamma_{G,t,T}$ versus Φ_G (similarly $\gamma_{G,t}$ versus Φ_G) fits results of Section 3.a. adequately for diverging lenses [14] but not for converging ones, the difference being 5.5% for 6 D (in Fig. 3 we show $\gamma_{G,t,T}|_{\text{far}}$, for simplicity leaving aside $\gamma_{G,t,T}|_{\text{near}}$ which is almost the same). Additionally, if D_G is almost zero then from Eq. (2) we obtain that visual magnification is almost unitary so if contact lenses are worn instead of ophthalmic ones, the eccentricity angle is almost the same as for the naked eye.

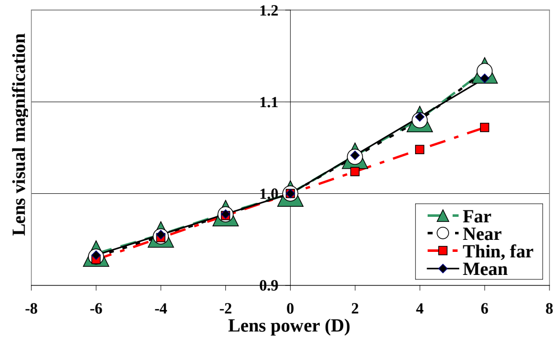

Fig. 3. Visual magnifications $\gamma_G|_{\text{far}}$; $\gamma_G|_{\text{near}}$; $\gamma_{G,t,T}|_{\text{far}}$ and Γ_G.

4. Formula proposed for the effective angle

If the data required to apply the method of Section 3.a (lens parameters, vertex distance and source location) are unknown and if the formula of Section 3.b does not yield accurate results, it is desirable to dispose of a formula for the visual magnification which is independent of the former data and fits paraxial ray tracing results better than Eq. (2). According to Fig. 3, γ_G depends almost linearly on Φ_G for $\Phi_G<0$ and also for $\Phi_G>0$ and it is nearly independent of source position. Taking this into account, we propose the following formula for the ophthalmic lens mean visual magnification, Γ_G,

$$\Gamma_G = 1 + a_G\Phi_G, \qquad (3)$$

where a_G is such that it is $a_G^{(M)}$ if $\Phi_G<0$ and $a_G^{(H)}$ if $\Phi_G>0$. We find these slopes first calculating the slopes $a_{Gr}^{(M)}$, $a_{Gr}^{(H)}$, $a_{Gp}^{(M)}$ and $a_{Gp}^{(H)}$ (respectively associated to the sets {far, $\Phi_G<0$}, {far, $\Phi_G>0$}, {near, $\Phi_G<0$} and {near, $\Phi_G>0$}) as the mean of the 3 values of $(\gamma_G\text{-}1)/\Phi_G$ obtained in Section 3.a. for ±6, ±4 and ±2 D. Averaging results shown in Fig.3 for far and near source, we have $a_G^{(M)}=(a_{Gr}^{(M)}+a_{Gp}^{(M)})/2$ and $a_G^{(H)}=(a_{Gr}^{(H)}+a_{Gp}^{(H)})/2$ and we get

$$a_G^{(M)} = 0.011D^{-1},\ a_G^{(H)} = 0.021D^{-1}. \qquad (4)$$

Hence the slope for converging lenses is almost twice that of diverging ones and we get $\Gamma_G^{(M)}$=0.93 if Φ_G=–6 D and $\Gamma_G^{(H)}$=1.13 if Φ_G=6 D (Fig. 3). The slopes of Eq. (4) have been obtained for the original set but, in general, both the slopes and their discontinuity, $\Delta a_G=a_G^{(H)}-a_G^{(M)}$, are affected by lens thickness (e_G) and vertex distance (D_G). To analyze the influence of e_G, we consider 2 sets of fictitious lenses with parameters identical to those of the original set except for the thickness, e_G being 8.2mm in one set and 2 mm in the other (Fig. 4(a)). For a given power, the mean visual magnification slightly depends on e_G for diverging lenses and, when e_G decreases, it tends to the value corresponding to a thin lens for converging ones. As expected, if e_G decreases then Δa_G decreases (Δa_G=0.017 D^{-1} if e_G=8.2mm and Δa_G=0.005 D^{-1} if e_G=2mm). Concerning the influence of D_G, the distance VC-image is such that, for a given power absolute value, $|s_C^{(M)}|>|s_C^{(H)}|$ (Fig. 2) and $|s_C^{(M)}|-|s_C^{(H)}|$ decreases if D_G decreases. When D_G decreases, Δa_G also decreases and for a 50% variation of D_G we get Δa_G=0.012 D^{-1} if D_G=18mm; Δa_G=0.010 D^{-1} if D_G=12mm and Δa_G=0.008 D^{-1} if D_G=6mm (Fig. 4(b)).

To analyze the precision with which Eqs.(3-4) predict the values of the visual magnification (γ_G) for vertex distances and lenses parameters different from the original ones, we consider the following:

i) For the original vertex distance, we consider the original and to 2 other sets of ophthalmic lenses (one with n_G=1.5 fabricated by Opulens and another with n_G=1.7). The axial thicknesses vary among sets but the difference between the visual magnification computed assuming thick lens and far (or near) source and Γ_G is less than 0.4% for diverging lenses and than 1.3% for converging ones (Fig.5).

ii) For the original set, we vary D_G in 50% (this variation being large compared to usual ones) and the mean visual magnifications corresponding to D_G=18mm and D_G=6mm differ from that evaluated for D_G=12mm (Eq.(3)) in less than 4.1% (Fig.4 (b)).

On the basis of these calculations, we estimate that for different source locations, ophthalmic lenses of up to ±6 D and habitual vertex distances (10 to 14 mm), visual magnifications computed using Eq.(3), deviate from paraxial results evaluated assuming the lens to be thick in no more than about 2% for diverging lenses and 3% for converging ones. Hence θ_G can be estimated with a reasonable error using the formula $\theta_G=\Gamma_G\theta$ with Γ_G given in Eqs.(3,4).

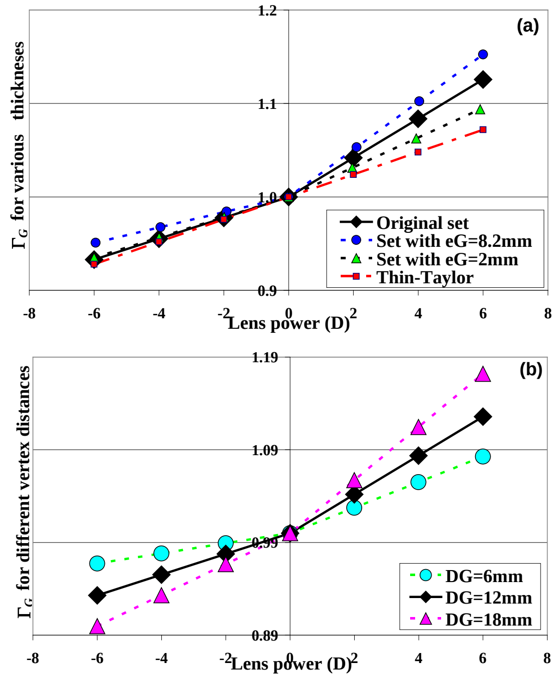

Fig. 4. Variation of magnification Γ_G with: (a) e_G, (b) D_G.

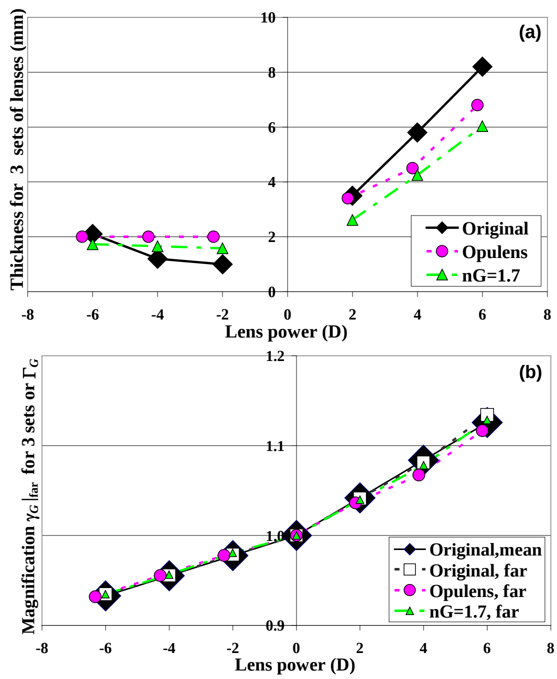

Fig. 5. Three sets of lenses. (a) e_G vs Φ_G. (b) γ_G vs Φ_G (the curve labelled "Original, mean" is the same as those labelled "Original set" in Fig. 4(a) and "Mean" in Fig. 3).

5. Application

As an example, we apply Eqs.(3,4) to one of the cases studied in a previous paper [10]. In that work, decrease of the perceived brightness of a foveal stimulus due to the presence of a peripheral glare source is measured in 1 emmetropic and in 3 myopic eyes wearing ophthalmic and contact lenses.

The subject sees two stimuli sequentially (Fig. 6(a) and 6(b)). The reference stimulus has luminance LR=1 cd/m^2 and is presented with glare yielding illuminance 15, 30 or 60 lx. The variable stimulus has luminance LX (with LX<LR) and is presented without glare, 6 different values of LX are used, each shown 52 times. In each trial, the subject´s task is to indicate the stimulus perceived as brighter. Each block of trials gives rise to a psychometric curve that enables the calculation of the matching luminance LM (value of LX perceived 26 times as being larger than LR). In Fig. 6(c) we show the limiting case, 60 lx glare illuminance and Φ_G=-4.25 D. The curve corresponding to the ophthalmic lens is to the left of that for the contact lens so $LM_G<LM_{LC}$. On the other hand, from Eqs.(2-4), the effective angle with ophthalmic lens is $\theta_G=\theta(1+a_G^{(M)}\Phi_G)$ = 9.5^o while the angle with contact lens is θ=10^o and, since glare affects matching luminance less when the eccentricity angle increases [9], we could predict $LM_G<LM_{LC}$. However, the lens also affects the illuminance reaching the eye and this could be accounted for [11] if lens parameters were known.

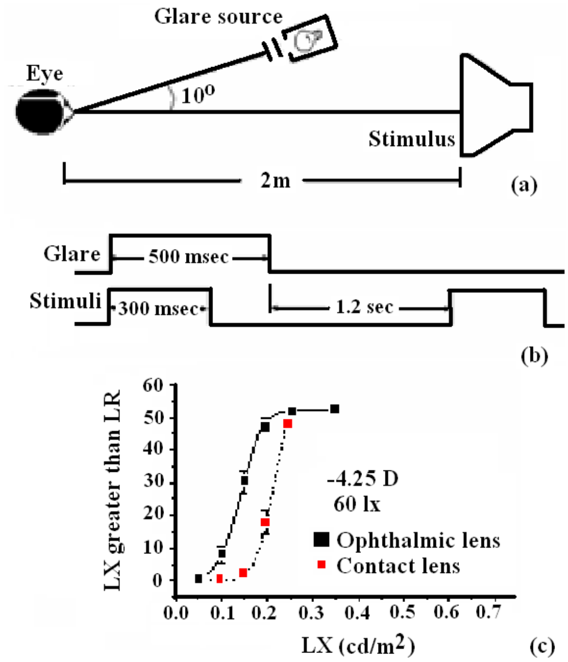

Fig.6. Brightness: (a) setup; (b) diagram; (c) curves.

6. Conclusion

If ametropic eyes wearing standard monofocal ophthalmic lenses look at a stimulus and receive light from a peripheral point source subtending an eccentricity angle θ at the corneal vertex, the effective angle, θ_G, is such that $\theta_G<\theta$ for myopes and $\theta_G>\theta$ for hyperopes. The angle θ_G depends on the ophthalmic lens power (Φ_G) and on other parameters and, under paraxial approximation, it can be computed considering the lens thick or thin. In the 1st case, configuration data (lens parameters, vertex distance and source position) must be known while, in the 2nd, the lens visual magnification (θ_G/θ) can be written in terms of Φ_G though for converging lenses this formula yields values which differ from those obtained in the 1st case, the difference being around 5.5% for a 6 D lens and a 12mm vertex distance. In the present paper we derive a formula which is specially useful if configuration data are not available and if the lens is converging. We approximate visual magnification for any standard ophthalmic lens to the mean one (Γ_G) calculated for an original set of lenses and for a 12mm vertex distance. The effective angle can be computed using the formula $\theta_G=\theta\Gamma_G=\theta(1+a_G\Phi_G)$ (with $a_G^{(M)}$=0.011 D^{-1} for diverging lenses and $a_G^{(H)}$=0.021 D^{-1} for converging ones). Values resulting from this formula do not always perfectly match those computed tracing paraxial rays but, for different source locations, ophthalmic lenses and habitual vertex distances, differences are up to about 2% for diverging lenses and 3% for converging 6 D ones.

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